

Rational Exponent Investigation KEY

Use a calculator. See definitions on last page.

1. Compare these sets of perfect squares affected by a square root or fractional exponent.

$9^{1/2} = 3$	$16^{1/2} = 4$	$25^{1/2} = 5$
$\sqrt{9} = 3$	$\sqrt{16} = 4$	$\sqrt{25} = 5$

If you applied a fractional exponent of $\frac{1}{2}$ and the square root sign to another set of perfect square numbers, what do you expect the result to be?

They would be the same.

Test two more sets of perfect square numbers and apply the fractional exponent of $\frac{1}{2}$ and the square root sign to them.

Many possibilities, but likely the next two perfect squares would be used:

$36^{1/2} = 6$	$49^{1/2} = 7$
$\sqrt{36} = 6$	$\sqrt{49} = 7$

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2. Compare these sets of whole numbers affected by a square root or fractional exponent.

$8^{1/2} = 2.8284$	$10^{1/2} = 3.1623$	$15^{1/2} = 3.8930$
$\sqrt{8} = 2.8284$	$\sqrt{10} = 3.1623$	$\sqrt{15} = 3.8930$

If you applied a fractional exponent of $\frac{1}{2}$ and the square root sign to another set of whole numbers, what do you expect the result to be?

They would be the same.

Test two more sets of whole numbers and apply the fractional exponent of $\frac{1}{2}$ and the square root sign to them.

Any number is acceptable, as long as the column pair are the same and the answer given for both in the column are the same and correct.

$\frac{1}{2} =$	$\frac{1}{2} =$
$\sqrt{\quad} =$	$\sqrt{\quad} =$

3. What pattern do you see emerging from #1 and #2?

Any number to the fractional exponent $\frac{1}{2}$ is the same as that number with the radical square root sign applied.

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4. Explain the pattern as a general mathematical rule, using the terms $x^{1/2}$ and \sqrt{x} .

$$x^{1/2} = \sqrt{x}$$

5. Based on the pattern discovered in #3 above, test different sets of whole number bases using **different** unit fractional exponents (suggested: $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc.) and **different radical indexes other than 2**. You will need to use a separate page.

Any comparison where the denominator of the fractional exponent and the index match, and the base numbers are identical.

Possibilities should look like the following:

$$27^{1/3} \text{ and } \sqrt[3]{27}$$

$$9^{1/3} \text{ and } \sqrt[3]{9}$$

$$8^{1/3} \text{ and } \sqrt[3]{8}$$

$$16^{1/4} \text{ and } \sqrt[4]{16}$$

$$243^{1/5} \text{ and } \sqrt[5]{243}$$

$$48^{1/4} \text{ and } \sqrt[4]{48}$$

$$32^{1/5} \text{ and } \sqrt[5]{32}$$

$$625^{1/5} \text{ and } \sqrt[5]{625}$$

$$81^{1/4} \text{ and } \sqrt[4]{81}$$

6. Can you create a general mathematical rule using $x^{1/n}$ and $\sqrt[n]{x}$ based on #5?

$$x^{1/n} = \sqrt[n]{x}$$

7. Based on the pattern discovered in #3 above, test different sets of whole number bases using **different** fractional exponents not using unit fractions (suggested: $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{3}{5}$, etc.) and **different radical indexes other than 2**. You will need to use a separate page.

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Any comparison where the denominator of the fractional exponent and the index match, and the base numbers are identical, including the numerator of the fractional exponent and the exponent applied to the number under the radical sign. The numerator of the fraction Possibilities should look like the following:

$$27^{1/3} \text{ and } \sqrt[3]{27}$$

$$9^{1/3} \text{ and } \sqrt[3]{9}$$

$$8^{1/3} \text{ and } \sqrt[3]{8}$$

$$16^{1/4} \text{ and } \sqrt[4]{16}$$

$$243^{1/5} \text{ and } \sqrt[5]{243}$$

$$48^{1/4} \text{ and } \sqrt[4]{48}$$

$$32^{1/5} \text{ and } \sqrt[5]{32}$$

$$625^{1/5} \text{ and } \sqrt[5]{625}$$

$$81^{1/4} \text{ and } \sqrt[4]{81}$$

8. Can you create a general mathematical rule? Use m as the numerator of the fractional exponent and n as the denominator of the fractional exponent.

$$x^{m/n} = \sqrt[n]{x^m}$$

Levels:

Limited - Filled in tables for #1 and #2 mostly correctly and predicted correctly.

Adequate - Completed #1-4 mostly correctly

Substantial - Completed #1-6 mostly correctly

Excellent - Completed #1-8 mostly correctly